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EXPERIMENTAL STUDY OF RAREFIED GAS FLOW BETWEEN PARALLEL PLATES

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EXPERIMENTAL STUDY OF RAREFIED GAS FLOW BETWEEN PARALLEL PLATES

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ABSTRACT. The flow of inert gases, hydrogen, and deuterium in a plane-parallel slot of height ~19 mm formed by glass plates is studied. A minimal value of the volumetric gas flow rate is observed for Kn ~ 1. The volumetric flow rate increases without limit for Kn > 1 in the pressure interval investigated. There is considerable difference in the experimental data for the different gases in the free molecular flow region. Anomalous behavior of neon in comparison with the other gases was noted. A comparison is made of the available theoretical results with the experimental data.

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Introduction

Rarefied gas dynamics has attracted the attention of researchers more and more recently. From the theoretical point of view, the problem lies in finding effective methods for the solution of the basic Boltzmann kinetic equation in application to any specific problem.

Unfortunately, there is no general method for the solution of this complex integro-differential equation. Therefore, it is very important to have experimental data, which in the final analysis must be used to resolve the question of the correctness of any particular solution procedure.

In this context, the study of gas flow between parallel plates for arbitrary Knudsen numbers is very useful. On the one hand, the theoretical solution of the problem is simplest for this geometry. On the other hand, the

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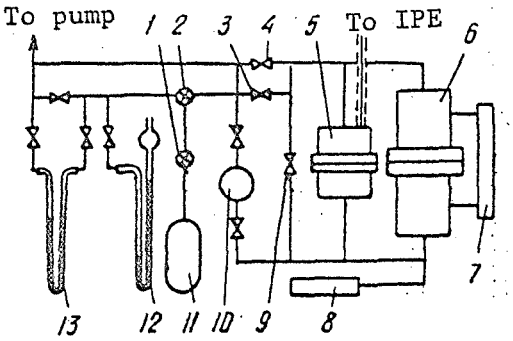

Figure 1. Main diagram of the experimental apparatus.

behavior of the corrected gas flow rate as a function of the Knudsen number has a characteristic singularity, first discovered experimentally by Knudsen [1] and Gaede [2]. The corrected flow rate has a minimum for Knudsen numbers of the order of one. Therefore, the theoretical procedures can be justified simply by the disclosure of this minimum, without resorting for the moment to extensive numerical calculations for a quantitative evaluation of the solutions. Unfortunately, at the present time no such experimental data are available other than Gaede's data, which the author himself admits are of a qualitative nature. There are also the relatively inaccessible data of Huang [3], whose scatter is so great that it is impossible to make any quantitative comparison.

In the following, we describe the experimental equipment and measurement technique. Experimental results obtained for glass plates for several gases are presented and discussed.

1. Experimental Apparatus

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A schematic of the experimental apparatus is shown in Figure 1. The test chamber 6 consists of two thin-walled brass shells with a volume of about 500 cm^3 each. Between the shells there is a brass flange in which glass plates forming the plane-parallel slot are mounted. The plates are made from K-8 glass with carefully ground and polished working surfaces, finished to class V accuracy (GOST [Government Standard] 11141-65). For purely practical reasons the plates were made with trapezoidal cross section. Aluminum foil about 20 microns thick was placed between the plates to form the plane-parallel slot.

A capacitive differential micromanometer 5 (pressure transducer) was used to measure the pressure difference in the experimental setup. The sensitive

element of this transducer consists of two beryllium bronze membranes, one of which is movable and provides vacuum-tight separation of the entire test section into the two volumes V_1 and V_2. The membranes form a variable capacitance condenser which is a part of the electrical circuit of the variable frequency generator of the IPE-2 capacitance increment gage. Since the construction and operating principle of the pressure transducer and the IPE-2 gage were described in detail in [4], we shall simply note some design characteristics of the transducer which we used.

Nitric acid was used to etch in the stationary membrane more than 200 holes of diameter 0.5-1 mm, whose total area exceeds by more than a factor of four the area of the circular slot formed by the membranes. The pressure transducer volumes were approximately the same on both sides of the movable membrane and equal to about 120 cm^3. The transducer is connected with the test chamber by copper tubing of about 7 mm diameter and very short length. The procedure for the fabrication of separate parts and components of the transducer was altered somewhat in order to increase its sensitivity by more than an order of magnitude in comparison with the previous construction.

The indicating instrument was a M198/2 millivoltmeter (class 1.0) which recorded the IPE-2 output voltage, proportional to the pressure transducer capacitance change. The capacitance increment gage together with the millivoltmeter has three operating ranges corresponding to the following frequencies: 0-200, 0-1023, and 0-4114 Hz. The millivoltmeter indications are linear with respect to frequency.

The pressure transducer was calibrated with the aid of the calibrating device 8, whose basic elements are a calibrated rod of diameter (0.595 ± 0.005) cm and a micrometer with rod position readout accuracy of 0.01 mm.

The device 7, consisting of a copper cylinder of diameter about 3 cm and length about 30 cm with polished internal surface, was used to maintain the /107
pressure differential across the ends of the slot. A rod with teflon seal moves inside the cylinder.

The working section of the experimental setup, including the pressure transducer, test chamber, the devices 7 and 8, the rotary bypass valve 9, and the fill 3 and evacuation 4 shutoff valves, was placed in a thermostat, consisting of a massive aluminum box with wall thickness 2 cm. The outside of this box was covered with foamed plastic having a thickness of about 5 cm. This thermostatting stabilized considerably the operation of the pressure transducer, which was very sensitive to variation of the ambient temperature.

The test gas was introduced from the tank 11 through the valve 1 with the aid of the three-way needle valve 10 and the shutoff valve 3. The gas was evacuated from the system by a VN-461M forevacuum pump and a VA-05-4 vacuum unit.

The absolute pressure in the test equipment was measured by manometers of three types. The conventional mercury manometer 12 was used for pressure measurements in the range 760-30 mm Hg. The absolute pressures in the range 30-5 mm HG were measured by the OM-30 optical manometer of accuracy class 1.0. For the measurements in the 10-0.2 mm Hg range, we used the liquid U-tube manometer 13, one leg of which was evacuated to about 10^{-5} mm Hg during the measurements. The liquid column height was measured by a KM-6 cathetometer. The indications of the optical and liquid manometers agreed in the pressure range where these indications overlapped to within the measurement error limits (about 1.5%). Absolute pressures below 0.2 mm Hg were measured by the McLeod compression manometer 10. The measurement error did not exceed 1.5% over the entire pressure range.

Vacuum tightness of the experimental setup was achieved by soldering the permanent joints using tin solder and the use of teflon seals in the disconnected joints. The pressure variation in the test section did not exceed $1 \cdot 10^{-4}$ mm Hg per hour.

2. Measurement Technique

The measurement technique was as follows. If the pressure in one of the volumes V_1 connected by a channel is increased by ΔP, the gas will flow across

into the volume V_2, and after the time interval t the pressures in the volumes equalize. We can assume that for the small time interval dt the gas flow in the channel is quasi-stationary, i.e., during this time the gas flows through the channel with a constant pressure differential $\Delta P'$. Then it is not difficult to obtain the expression for the quantity of gas M, calculated from the product of the pressure and the volumetric flow rate, which flows through the channel per unit time for unit pressure gradient and a given average absolute pressure $\overline{P}$, which is defined by the following formula

$$M = \frac{1}{t}\frac{V_1 V_2}{V_1+V_2}\left[1+a\overline{P}\left(\frac{1}{V_1}+\frac{1}{V_2}\right)\right]\ln\frac{\Delta P_1}{\Delta P_2}.$$

This is the computational formula for finding the experimental volumetric gas flow rate in a rectangular slot. In this formula ΔP_1 and ΔP_2 denote the pressure differences at the ends of the slot, measured over the time interval t. The quantity a is a constant for a given experimental setup and denotes the volume change caused by deflection of the movable membrane of the pressure transducer for a change of the average pressure by 1 mm Hg and equals 0.135 cm^3/mm Hg.

The pressure transducer sensitivity was measured with the aid of the calibrating device 8. /108 We determined the dependence of the millivoltmeter indications Δn on the pressure difference ΔP which occurs with displacement of the calibrating device rod through the distance Δl. The measurements showed that this relation is linear for all three operating frequency ranges. Therefore, in the computational formula the ratio $\Delta P_1/\Delta P_2$ can be replaced by the ratio $\Delta n_1/\Delta n_2$ of the number of millivoltmeter divisions, measured over the time t. The pressure transducer sensitivity, defined by the ratio $\Delta P/\Delta n$, was $(4.06 \pm 0.03) \cdot 10^{-5}$ in the first range, $(2.02 \pm 0.01) \cdot 10^{-4}$ in the second range, and $(8.08 \pm 0.05) \cdot 10^{-4}$ mm Hg per division in the third range.

The volumes V_1 and V_2 were measured from the known control volume with account for the correction for the nonideality of the gas and were equal to $V_1 = (916 \pm 3)$ cm^3, $V_2 = (997 \pm 4)$ cm^3.

Prior to entry of the test gas, the experimental setup was evacuated by the vacuum system for a period of 3-4 hours. At the same time the working section of the setup was heated by a gasoline burner to a temperature of about 80-100° to accelerate scavenging of the gas adsorbed by the walls. After filling with the test gas, the setup was allowed to stand for 0.5-3 hours. Then the volumetric gas flow rate M was measured. To do this, the bypass valve 9 was closed and the device 7 was used to create the given pressure differential ΔP between the volumes V_1 and V_2 or Δn, and this differential was maintained by movement of the rod of the device 7 during a period of 20-30 sec for high absolute pressures and for 60-90 sec for low pressures. A stopwatch was started at the same time. After this soaking, simultaneous readings were made of the millivoltmeter Δn_i and the stopwatch t_i. The test was continued for 40-50 sec at the high pressures and up to 7-8 min in the intermediate flow region at pressures of 0.5-7 mm Hg. The experiment was terminated after Δn decreased by 1/2 to 2/3 of the initial value. An EPP-09 automatic recording potentiometer was used to monitor the millivoltmeter indications. An exponential nature of the variation of Δn with time was observed over the entire pressure range tested.

The gas volumetric flow rate was calculated using the formula presented above for the ten largest time intervals Δt_i. The average deviation of M did not exceed 1.0%. for the high and moderate pressures and 1.5% for pressures less than 0.1 mm Hg. The magnitude of the gas volumetric flow rate M measured for a given absolute pressure in different frequency ranges remained constant within the experimental accuracy limits, which did not exceed 1.5% in the minimal volumetric flow rate region.

The water vapor present in certain of the gases was frozen out in a trap cooled by liquid nitrogen. After freezing-out, the gases were passed through a trap filled with phosphoric anhydride. In the purified gases, the water vapor content level did not exceed the background level of the mass spectrometer used to verify the gas purity. The purity of the test gases was at least 99%.

The slot height was measured by an interference method using a laser of the Lak-1 type from a determination of the angle between the neighboring bands

which occur with interference of the laser rays reflected from the surfaces of the slot. The measurements showed that the slot height d was (19.0 ± 0.3) microns. For a check the slot height was measured on an optical microscope, which gave a value of (18 ± 1) microns. The slot length l and width b were: /109
l = (0.805 ± 0.002) cm, b = (3.346 ± 0.002) cm.

3. Experimental Results

We studied flow in the plane-parallel slot of hydrogen, deuterium, and inert gases for average absolute pressures in the range from 760 to 10^{-3} mm Hg. In the viscous slip flow region the Poiseuille formula, accounting for the gas slip at the wall, was used to calculate by the least squares method the slot height d and the slip constant σ. The mean free path λ was determined from the well-known viscosity formula for hard spherical molecules.

The table shows the calculated values of d and σ with the standard deviations for the gases tested. The average slot height (18.86 ± 0.13) μ for all the gases was used in the subsequent calculations. In the calculations, we took into account the end-effect corrections associated with the finite width of the slot and the kinetic energy dissipation at the ends of the channel. In view of the low flow velocities, the corrections due to the process of velocity profile formation in the channel and the energy dissipation due to the viscous forces were negligibly small.

TABLE

Gas	H_2	D_2	He	Ne	Ar	Kr	Xe
σ	1,44	1,49	1,46	1,52	1,43	1,36	1,37
Δσ	0,03	0,04	0,02	0,02	0,04	0,04	0,03
d (μ)	18,77	18,80	18,84	18,92	18,93	18,85	18,92
Δd (μ)	0,11	0,14	0,11	0,09	0,14	0,16	0,13

We see from the table that the slip constants σ differ for the different gases, and this difference for the heavy gases exceeds the experimental error

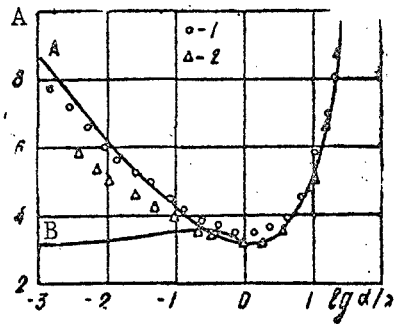

Figure 2. Corrected gas volumetric flow rate as a function of inverse Knudsen number. 1 — Helium; 2 — Xenene.

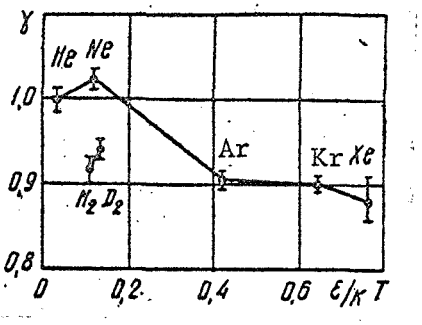

Figure 3.

limits and exceeds by 10-15% the value of σ obtained experimentally by Knudsen in circular glass capillaries [5] and by Lund and Berman [6] in circular metal and crystal capillaries.

Figure 2 shows the corrected gas volumetric flow rate Q determined from the ratio of the experimental value of the volumetric flow rate M of the complex $\frac{1}{4}(2kT/m)^{1/2}(bd^2/l)$ versus the Knudsen number d/λ for helium and xenene. For the other gases, the experimental values of Q were located between these limiting curves. We observe a marked minimum of Q in the intermediate flow region for $d/\lambda \simeq 1$. While in the viscous slip flow region ($d/\lambda > 10$) the experimental values of Q are located practically along a straight line, in the intermediate and free-molecular flow regions we observe a divergence of the experimental curves for Q. In order to evaluate the magnitude of this deviation with respect to the values of Q for helium, we drew graphically a curve relative to which we calculated the quantity γ, equal to the ratio of the experimental value of Q for the given gas to the graphical value of Q_g for the same $d/\lambda < 0.5$.

The average values of γ with the corresponding error are shown in Figure 3. /110 The abscissa is the value of ε/kT for the test gases, where ε is the potential interaction parameter [7] and T is the average experimental temperature (T = 295°K). We note a significant difference in the value of γ for the heavy inert gases in comparison with He and Ne and an anomalous value of γ for neon and

deuterium. The slip constants σ shown in the table have a similar variation with ε/kT.

For purposes of comparison, Figure 2 shows the theoretical curves for the corrected gas volumetric flow rate Q. The curve A was obtained by Cercignani in the solution of the model equation by the integral method [8], while we obtained curve B for Maxwellian molecules using the method described in [9]. Agreement of the experimental and theoretical values is observed in the viscous slip flow region. Curve A also describes the experimental data quite well in the free-molecular flow region, while in this region the curve B indicates slow convergence of the moment method for the solution of the Boltzmann equation. This circumstance indicates the need for further work to find effective methods for the solution of the kinetic equation.

Conclusions

A measurement technique has been worked out and an experimental study has been made of the flow of several gases in a plane-parallel slot. The slip constants σ, whose value depends on the type of gas, have been calculated.

In the intermediate flow region for Kn $\simeq$ 1 there is a minimum of the corrected volumetric gas flow rate. With increase of Kn we observe an unlimited increase of the gas volumetric flow rate (in the pressure range studied) and a significant difference of the experimental data for the different gases. We note anomalous behavior of neon in comparison with the other inert gases and of deuterium in comparison with hydrogen, similar to that noted in [6].

This deviation of the experimental data may more likely be explained by the different nature of the interaction of the gases with the slot surface (namely, the molecule lifetime at the wall) than by wall roughness or the different fraction of the molecules reflected specularly by the wall. However, this argument requires additional experimental study.

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